Tim Stinchcombe - Tri to sine conversion

	**!! Under Construction !!** Page is very draft and contains much guff that needs fleshing out!	

Triangle wave to sine wave conversion

I find the idea of fashioning a sine wave from a triangle wave by passing it through some sort of nonlinear waveshaping device a fascinating one, especially since many of these devices, superficially at least, seem to bear no relation whatsoever to a sine function. Some years ago I started documenting references whenever I came across a new one, and the list has now grown pretty large. I thus thought I would make a page dedicated to this subject: I'm not quite sure just what format it will eventually take, as I'm a little hesitant about copying lots of schematics from many books, and thus how useful it will be, as without doing so, the page may seem a little *effete*. It is thus little more than a long list of references and links, strongly biased toward such circuits as used in synthesizers, which is where my main interest lies.

I have split the page into three: *piecewise linear*, 'PWL', for approximations using multiple linear segments; *overdriven differential pair*, 'ODP', for those utilizing the standard bipolar differential pair, including those using operational transconductance amplifiers (OTAs); and *Middlebrook & Richer*, 'MR', for those using the nonlinear characteristics of a JFET, such circuits almost exclusively being derived from their 1965 paper. I have categorized references into: journals; synthesizer-related books; other books (including datasheets etc.); and links—though there is naturally some cross-over between these. As there are so many references in *Electronotes*, they get their own section at the end.

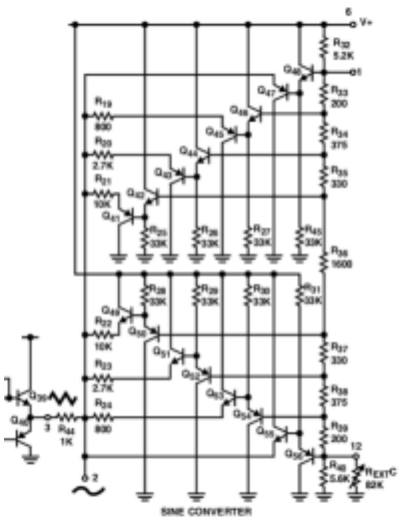
I will also point out that whilst I have all the papers and books mentioned, I have only practical experience of a very few of these circuits, and so it should be clear that my interest *is* more on the academic side—I am *not* on a crusade to find the absolute most accurate, bestest of all-time, tri-to-sine converter! However I do hope there is something here of use to others, to entertain and to inform!

Please note that not all links on book or journal paper titles are going to take you to a *free* downloadable copy of that reference—it will for some of them (I lucked out on a couple of the papers!), but for others, notably the Thomas Henry books where these are still available, they take you to a place where you may purchase the book, normally for a reasonable fee. Many of the references are from old books, long out of print (I have been building my collection over several years), and my copies of the papers are mostly obtained through the 'official' channels, i.e. a document request to the British Library via my local town library, and I have no intention of breaching the copyright notice I sign each time I get one (by for example scanning them in their entirety and posting them online...).

[The main focus of this page is to provide information on sine wave approximation using *nonlinear* devices, and so I am going to completely ignore other methods, such as digital or even polynomial approximation (which can require lots of expensive multipliers). If you don't see your favourite one listed, or if you feel I have left something important out, then please send me the details and I'll consider including it, especially if it is a clear schematic online that I can link to.]

Piecewise linear approximation

This is probably the oldest method—here resistors with diodes and/or transistors are arranged to approximate a sine curve with a series of straight-line segments. Many different types of arrangements are possible, but an obvious disadvantage is that to accurately model the sine wave, many linear segments are required, hence requiring many components, and soon we encounter difficulties matching all the component values etc. An example of the complexity such a scheme can reach is the (now obsolete) ICL8038 function generator chip, of which, here is the equivalent schematic of the sine converter:



Being fabricated in a monolithic chip, matching between the transistors is probably quite good, but I do not know how accurately they needed the resistors—THD is quoted as "close to 0.5%" when external pots are used for trimming.

Journal refs

Monolithic Waveform Generation
AB Grebene, IEEE Spectrum, 9, pp34-40, Apr 1972
This is really a breakdown of how a function generator can be implemented on a monolithic chip (specifically the XR-205), but in passing it briefly mentions two methods of tri-to-sine waveshaping: the piecewise linear method using diode-resistor networks, and overdriving a differential pair.

Synth-related book refs

Electronic Music Circuit Guidebook

B Ward, Tab Books, 1975, p175

A rather complicated-looking PWL circuit based around an LM3900, delivering "a reasonably good sine wave".

Electronic Musical Instruments

N Crowhurst, Foulsham-Tab Ltd, 1975, p134

A simple transistor/diode PWL circuit, "correctly adjusted, can produce a sine wave with much less than 1 percent total harmonic content", yet there are no trimmers in sight, so one is pretty much on one's own as to how this is achieved!

Electronic Music Circuits

B Klein, 1995, p96

(For details of the PWL circuit on p96, see the main entry below.)

Other book refs

IC and Waveform Generator Handbook

RM Marston, Newnes, 1990, p137

A simple diode-based PWL circuit, giving typically less than 2% THD.

Oscillator Circuits

RF Graf, Newnes, 1997, p119 & p94/95

Page 119 is the PWL circuit figure 12 of the CA3140 datasheet immediately below (it refers to an older GE/RCA document of 1987). Two copies of the MR circuit (using 2N4093 JFETs) appear on pages 94/95, apparently culled from *Electronic Engineering* magazine, Sept 1984, p37—claimed THD less than 1%. (The book is merely a collection of schematics from datasheets etc.).

CA3140 4.5MHz BiMOS Operational Amplifier with MOSFET Input/Bipolar Output

Intersil datasheet, July 2005, p10 & 12

Figure 12 shows a sine wave shaper using a tricky-looking arrangement of diodes in a CA3019 diode array. I've not examined it in any detail, but as the text mentions "two break points are established..." I'm assuming it is a PWL-type of circuit, and include it in this section. Quoted THD typically less than 2%.

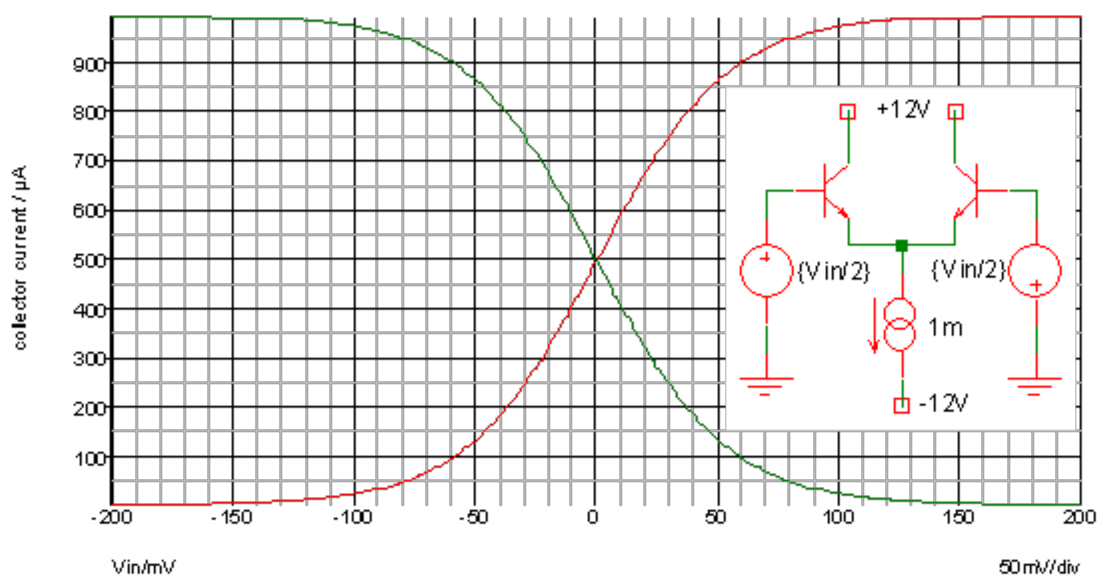
Sine Wave Generation Techniques

Application Note AN-263, Texas Instruments, April 2013, p10

(For details of the PWL circuit on p11, see the main entry below.)

Overdriven differential pair approach

probably most popular; makes use of the 'bow-tie' curve that arises as the differential voltage across pair increases.
three ways: diff pair; diff pair + emit degen; plus feedforward, cusp cancel
genealogy: diff pair used, Meyer et al formalize, Evans&Schiffer get wind of cusp cancelling idea, though clearly already in use -- Buchla early 70s, Aries 317 VCO Dennis Colin
needs careful assignment of values to minimise THD; may add more later if time permits



does anti-parallel diodes (a la Doepfer) fall into this category?

Journal refs

Accurate Triangle-Sine Converter

G Klein & H Hagenbeuk, Electronic Engineering, 39(477), pp700-704, Nov 1967

This is a kind of 'hybrid' of other methods. Here a differential pair is fed with antiphase triangle waves, and the emitter circuit contains a resistor-diode network whose conductance alters with the amplitude of the input triangle wave; the shaped sine wave then appears as the differential signal at the collectors. It is claimed to be insensitive to the tolerances of the resistances used, and is thus suitable for large scale integration. Measurements from a simple (discrete) implementation gave 0.2% THD at low frequencies up to 0.8% at several hundred kilohertz. (This appears to be the full paper of a conference paper of the same title by the lead author, submitted to the 1967 Int'l Solid-State Circuits Conference.)

New Approach to Triangle-Sine-wave Generation

C Paull & Evans WA, Electronics Letters, 6(19), pp620-621, Sep 1970

This uses a different approach from the normal overdriven differential pair of most of the papers in this section. It uses a number of differential pairs with emitter degeneration to produce clipped triangle waves at the outputs; by carefully choosing the clipping points, and summing across all the pairs, a piecewise linear approximation to a sine wave is produced. With a 7-segment approximation they achieved a THD of 0.2%. (No practical circuit is given.)

The Differential Pair as a Triangle-Sine Wave Converter

RG Meyer, WMC Sansen, S Lui & S Peeters, IEEE J SSC, 11(3), pp419-420, Jun 1976

The introduction of this paper seems to suggest that Grebene was the first to suggest using a differential pair for triangle-sine conversion, and in the paper they put this idea into a more formal framework **includes emitter degeneration resistor**. Computed and measured THDs agree, down to a value of about 0.2% (the experimental circuit is not given).

A Low Distortion Tri-wave to Sine Converter

WA Evans & V Schiffer, Radio and Electronic Engineer, 45(5), pp217-224, May 1977

introduces 'cusp cancelling', in an overdriven diff pair circuit; simple algebra shows why cancelling works; claimed THD 0.05% or less, but a lot depends on purity of input tri wave!; strongly suspect this was putting the known technique of subtracting out a portion of the triangle wave from the converted sine, in order to remove the 'cusp', on firmer academic footing; quite sure technique know before 1977, e.g. Aries 317 VCO, and in context of FET-based MR shaper, Buchla VCOs (early 70s)

Canceling Cusps on the Peaks of Shaped Sine Waves

SH Burns, Electronics, p110, Jul 1982

A rather unsatisfactory single-page article, in as much as it shows the cusp-cancelling method in general, and specifically as part of an overdriven differential pair arrangement, and compares the THD in tabular form against other methods, except the exact arrangement and source/references of those arrangements is not given!

Synth-related book refs

(Several of the Thomas Henry books have been made available again through Magic Smoke Electronics.)

VCO Chip Cookbook

T Henry, SMS Electronics, 2007, pp26-29

Similar sine shaping circuits appear in several of the Thomas Henry books: this one contains two, and probably has a more in-depth description of them than the others. first is commoner diff pair, emitter res, others calls feedback; second entitled 'ultra-precise sine shaper' is 3080-OTA THD 0.1%, gets rid of 'pips'/cusps present in other methods feedforward resistor, subtracts out a fraction of tri wave

Build a Better Music Synthesizer

T Henry, Tab Books Inc, 1987, p60

Diff pair method, same Deluxe VCO as Polyphony reprint, feedback resistor (incorrectly named as R3 in text, appears as R1 in schematic Fig 5-2,

Electronic Music Circuits: The Reprints Volume I

T Henry, Midwest Analog Products, 2002, p76 &

two circuits here too; first p76 reprint of Polyphony Nov 1981 CEM3340-base 'Deluxe VCO', same diff pair-based shaper with feedback resistor between emitters ('R3'). Second is Apr Polyphony 1983, diff pair and feedback resistor

Electronic Drum Cookbook

T Henry, Midwest Analog Products, 2002, p33

same circuit as VCO book, slight diffs values, feedforward for cusp cancelling

An Analog Synthesizer for the 21st Century

T Henry, Magic Smoke Electronics, 2006, p13

A collection of schematics and parts lists—there is no explanation or chit-chat here, that *is* it—much of which is doubtless similar to his previously published circuits. The LM13700-based 'LM VCO' on page 13 includes the same diff pair-based shaper (with emitter degeneration resistor) as several of those above.

Electronic Music Circuits

B Klein, 1995 (originally Sams, 1985, now self-published), p91 et seq

Fig 4-7, p91 is *Electronotes* ENS-76 VCO Option 2, with a CA3080-based ODP; Fig 4-9, p94 is *Electronotes* ENS-76 VCO Option 1, with an MR circuit using an MPF-102 JFET; Fig 4-10, p96 shows a PWL circuit; Fig 4-11, p97 is the CA3280 dual OTA ODP-type circuit; Fig 4-12, p98 is a CA3080-based ODP circuit, but is one of the few examples utilizing the 'cusp cancelling' resistor (18kΩ on Q1 collector; also note error at '−' op amp input...)

Other book refs

IC Op-Amp Cookbook

WG Jung, Howard Sams Inc: 1st edn, 1974, §7.4.4, p386
3rd edn, 1997, §10.3.4, p479

Fairly standard ODP using a matched pair (LM114); the third edition version uses a CA3096 transistor array, with optional temperature oven for temperature stability, with claimed THD less than 1%, or 0.3% when used with offset nulling and selecting the device for close matching; third edition also gives a circuit using a trig function generator IC, the AD639.

IC Array Cookbook

WG Jung, Hayden Book Co Inc, 1980, p121

ODP using a CA3046 transistor array; THD of less than 1%.

IC Timer Cookbook

WG Jung, Howard Sams Inc: 2nd edn, 1983, p242

ODP using a CA3046 transistor array: with less trimming, THD is 1 to 2%; with additional trimming, a claimed 0.3% is possible.

CA3280 Dual 9MHz Operational Transconductance Amplifier

Intersil datasheet, May 2002, p5

Figure 10 on page 6 shows an ODP-type arrangement using both OTAs in a CA3280 package—they are doing something clever with the emitters to ensure the amplifiers cut-off completely at the peak of the triangle, which presumably helps reduce the little 'cusps' at the peak of the sine wave output. Claimed THD 'approximately 0.37%'.

Sine Wave Generation Techniques

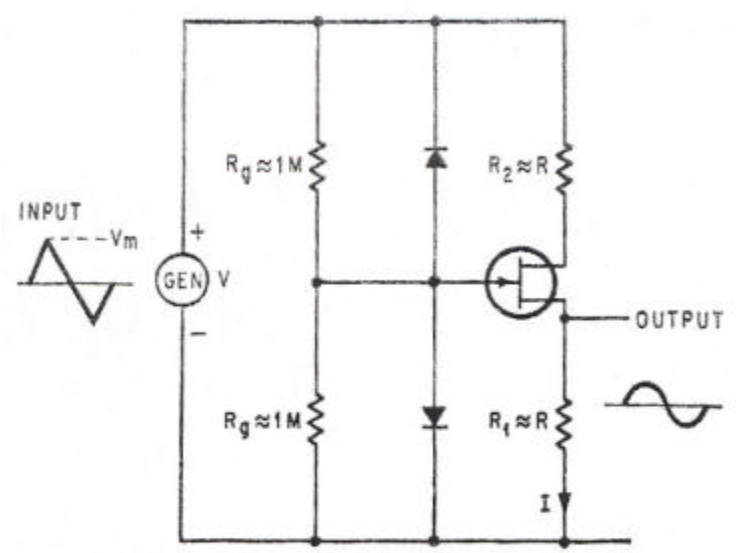
Application Note AN-263, Texas Instruments, April 2013, p10 et seq

As well as describing various sine wave oscillator circuits, this application note also has a section on 'Approximation Methods', including a fairly typical PWL circuit, Figure 12 on p11 (note it specifies E96-series resistor values); same ODP part of Figure 14 p13 & Figure 18 p16

Middlebrook & Richer JFET-based method

somewhere need Buchla links, and how this is a hybrid of MR + cusp cancelling - no knowledge of whether this actually helps, as clearly as it does for the BJT case...

The Middlebrook & Richer technique uses the nonlinear properties of a JFET to produce the sine-shaping, using the JFETs symmetry in an elegant and very distinctive configuration:



It was popular in many older synth circuits; to maintain accuracy, the amplitude of the input triangle wave needs to be well controlled, and this needs to be in the correct relation to the chosen JFET's threshold voltage in order to ensure that the JFET does indeed cut-off at the top of the triangle; the original paper also says one of two resistors can be trimmed, but implementations of the circuit are rarely seen with this.

Journal refs

Nonreactive Filter Converts Triangular Waves to Sines

RD Middlebrook & I Richer, Electronics, 38, pp96-101, Mar 1965

By simple adjustment of the input triangle amplitude, and one of the resistors for second harmonic distortion, the paper claims total distortion can be 'reliably maintained at under 1%' (and apparently this is relatively independent of temperature too). I did however have some difficulty replicating the algebra, and it looks to me like the key ratio of V_m/V_p doesn't follow: for further comments, refer to those I made whilst modifying one of my A-110 oscillators.

Low-distortion Triangle to Sine Conversion

MJ Declercq, Electronic Engineering, 45, p19, Oct 1979

Slight reworking of the MR idea - FET circuit hooked-up as input to op amp i-to-v config

Field Effect Transistor Converts Triangles to Sines

WE Peterson, Electronics, 43(18), pp69-70, Aug 1970

This is little more than a practical circuit of the MR method; THD 'of less than 2% is easily achieved'.

FET Differential Amplifier as a Tri-Wave to Sine Converter

H Hassan, Proceedings of 36th Southeastern Symposium on System Theory, pp427-430, 2004

This is a FET-equivalent of the overdriven (BJT) differential pair circuit above. A discrete example circuit (using 2N4860 JFETs) produces a claimed THD of 0.05%, with the possibility of perhaps another order of magnitude improvement (due to a wiring problem with the prototype).

Synth-related book refs

Musical Applications of Microprocessors

H Chamberlin, Hayden Book Co, 2nd edn, 1985, pp192-195

Shows the basic MR set-up, stating less than 1% THD is usually possible; a complete practical circuit as part of VCO is also given, which includes shape and symmetry trim.

Electronic Music Circuits

B Klein, 1995, p94
(For details of the MR circuit on p94, see the main entry above.)

Other book refs

Junction Field-Effect Transistors
CD Todd, John Wiley & Sons, 1968, p86
Shows two of the basic bridge circuits of the MR technique, to illustrate how it works, claiming 0.35% THD for the best arrangement, but no practical circuit is given.

Converter and Filter Circuits
RF Graf, Newnes, 1993, p118
Excerpt from Signetics AN186 immediately below (the book is merely a collection of schematics from datasheets etc.).

AN186 Waveform Generators with the NE566 in *Linear Data Manual, Volume 1: Communications*
Signetics, 1987, p4-311
This short application note contains an example of the MR circuit, albeit uncharacteristically using a *p-channel* JFET, as normally an n-channel one is used (this requires a reversal of the other diodes in the arrangement); distortion of 'less than 2%' is quoted. The circuit diagram is here at the SeekIC forum, along with a further reference to another Signetics document.

Oscillator Circuits
RF Graf, Newnes, 1997, p94/95
(For details of the MR circuit on p94/95, see the main entry above.)

Electronotes refs

Not surprisingly there are a lot of references in *Electronotes*, so I have given them a section of their own. (See my notes on the main synth page for what they are, and also various indices of them.) Since there are so many references, I dare say I have probably missed some! ('#X (Y)' refers to 'page Y in issue X'.)

#7 (6): 'FET Waveshaper'
Outline MR circuit—no practical values or device types given.

#28 (6): 'ENS-73 VCO Option 3'
Practical MR circuit given as part of overall VCO, using 2N3819.

#36 (5): 'ENS-73 VCO Option 2'
MR subcircuit same as #28(6) (the VCO is different).

#39 (6): 'On Converting Triangle to Sine'
Refinement to #28(6) & #36(5): removing an op amp—MR part of circuit stays same.

#55 (19): 'A New Sinewave Shaper applied to a Sweep Generator Design'
ODP using a CA3080, followed by a tracking filter; 1% THD figure attributed to both MR and ODP methods.

#75 (7): 'ENS-76 VCO Option 1'
Standard MR method using an MPF-102 JFET; comment that MR used rather than ODP "because it gives a slightly better looking waveform", and because a sufficiently large ±10V triangle is available to drive it into the nonlinear region.

#75 (18): 'ENS-76 VCO Option 4'

#75 (20): 'ENS-76 VCO Option 5'
ODP using a CA3080, plus *two* tracking one-pole filters! No distortion figure given, only "Our setup gave us an excellent sinewave from 30Hz to 30kHz".

#82 (5-17): 'Mathematical Analysis of Differential Amplifier Triangle-to-sine Converters'
A thorough look at the ODP method with tied emitters, i.e. *without* an emitter degeneration resistor, though that gets a brief mention at the end. Claimed THD for these circuits is around 0.2%; it's a shame that this was apparently published so close to the Evans & Schiffer paper which introduced the 'cusp cancelling'—it would have been nice to see some thoughts on that!

#122 (3 & 20): 'Examination of Distortion Remaining from Common Triangle-to-Sine Converters'
A test circuit for examining the distortion present in a sine shaper—a CA3080-based ODP example is used.

#123 (3): 'More Tests of Triangle-to-Sine Converters'

A continuation of #122(20), this time with an MR circuit using an MPF-102 JFET. The FET way gave an error about half of that using the OTA (figures are not THD, so I won't quote them); it notes that the OTA method produces minimum audible distortion when the waveform is a little pointy at the top, and that the FET method has a better looking shape!

#142 (4): 'A new Triangle-to-Sine Converter using a 1496-type Multiplier'

An ODP circuit using an MC1496 balanced modulator chip—almost literally just the circuit, with minimal chat (the biasing of the upper 'carrier' transistors looks a little odd to me, but I've not studied it in detail...).

AN-115: 'Triangle-to-Sine Converters using Nonlinear Shapers'

A very brief look at sine shaping, covering both the MR and ODP methods, stating less than 2% THD achievable for either method (circuits given are just about practical, though the JFET for the MR circuit isn't specified); two PWL methods are also briefly mentioned, but no practical circuits are given.

AN-348 (4): 'Contrasting Sinewave Generation in the Analog and Digital cases'

A CA3080-based ODP really only mentioned in passing.

Musical Engineer's Handbook

B Hutchins, 1975, Ch 5b, pp18-21 & p25

In the chapter on 'Voltage-Controlled Oscillator Design', a circuit for the MR method is simply given (using a 2N3819), with little description; there is a little more discussion on the ODP method, with a few equations and graphs, and a CA3080-based example is given.

[Page last updated: 01 Mar 2015]